الأزهر الشريف
قطاع المعاهد الأزهرية

نموذج رقم (١)

نموذج إجابة لامتحان الشهادة الثانوية الأزهرية

للعام الدراسي ١٤٤١هـ - ٢٠١٩ / ٢٠٢٠م

الدور الثاني

القسم : العلمي(نظام حديث)

مادة : التفاضل والتكامل(مترجم)

عدد الأسئلة (٥)

علماً بأن النموذج استرشادياً

Q1

(3 marks)

	Solution	mark
1	$\frac{1}{y^3}$	$\frac{1}{2}$
2	-6	$\frac{1}{2}$
3	9	$\frac{1}{2}$
4	$\{2, 6, 3\}$	$\frac{1}{2}$
5	2	$\frac{1}{2}$
6	2	$\frac{1}{2}$

Q2 (a) at $x = \frac{\pi}{4}$ $\therefore y = 3 - \cot^2\frac{\pi}{4} = 2$

$\therefore$ The point is $\left(\frac{\pi}{4}, 2\right)$ $\frac{1}{2}$

$y' = 2(-\cot x)(-\csc^2 x) = 2\cot x \csc^2 x$

$\therefore m = 2\cot 45^\circ \times \csc^2 45^\circ = 4$ $\frac{1}{2}$

Equation of the tangent

$y - 2 = 4\left(x - \frac{\pi}{4}\right)$ $\therefore 4x - y + 2 - \pi = 0$ $\frac{1}{2}$

Equation of the normal

$y - 2 = \frac{-1}{4}\left(x - \frac{\pi}{4}\right)$ $\therefore x + 4y - 8 - \frac{\pi}{4} = 0$ $\frac{1}{2}$

(2 marks)

(b) 1) $\int_0^1 \frac{3e^x - 2e^{2x}}{2e^x}\,dx = \int_0^1 \left(\frac{3}{2} - e^x\right)dx$ $\frac{1}{2}$

$= \left[\frac{3}{2}x - e^x\right]_0^1 = \frac{5}{2} - e$ $\frac{1}{2}$

2) $\int \frac{(3x-1)^2}{3x}\,dx = \int \frac{9x^2 - 6x + 1}{3x}\,dx$

$= \int \left(3x - 2 + \frac{1}{3x}\right)dx$ $\frac{1}{2}$

$= \frac{3}{2}x^2 - 2x + \frac{1}{3}\ln|x| + c$

$, x \neq 0$ $\frac{1}{2}$

(2 marks)

Q3

(a) differentiate with respect to x

$2y \times \frac{dy}{dx} + 2x \times \frac{dy}{dx} + 2y = 0$ $\frac{1}{2}$

differentiate again with respect to x

$y \times \frac{d^2y}{dx^2} + \frac{dy}{dx} \times \frac{dy}{dx} + x \frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$ 1

$\therefore (x+y)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 = 0$ $\frac{1}{2}$

(2 marks)

(b)

$\because \frac{dy}{dx} = 2 + \frac{3}{x}$

$\therefore \int dy = \int (2 + \frac{3}{x})dx$ $\frac{1}{2}$

$\therefore y = 2x + 3\ln|x| + c$ $\frac{1}{2}$

at the point $(e, 2e+5)$

$\therefore 2e + 5 = 2e + 3\ln e + c$

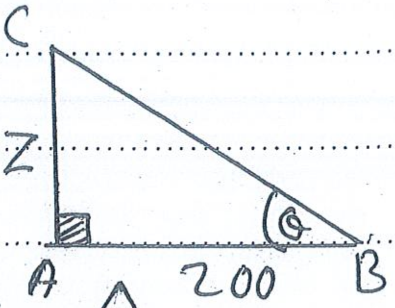
$\therefore c = 2$ $\frac{1}{2}$

$\therefore$ The equation of the curve is

$y = 2x + 3\ln|x| + 2$ $\frac{1}{2}$

(2 marks)

Q4 (a)



$\therefore \tan\theta = \frac{Z}{200}$ (1/2)

$\therefore \sec^2\theta \frac{d\theta}{dt} = \frac{1}{200}\frac{dZ}{dt}$ (1/2)

$\therefore \sec^2 45^\circ \times 0.12 = \frac{1}{200} \times \frac{dZ}{dt}$ (1/2)

$\therefore \frac{dZ}{dt} = 2 \times 0.12 \times 200 = 48$ rad/min (1/2)

(2 marks)

(b)

1) put $y = \ln x$ $\quad dz = x^2 dx$

$dy = \frac{1}{x} dx \quad z = \frac{x^3}{3}$ (1/2)

$= \frac{x^3}{3}\ln x - \int \frac{1}{x} \times \frac{x^3}{3} dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$ (1/2)

(2 marks)

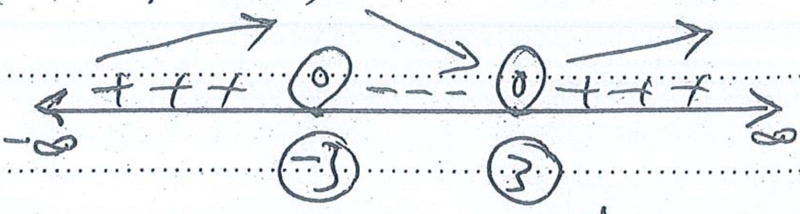
2) $\int_0^2 -(x-2)dx + \int_2^5 (x-2)dx$

$= \left[\frac{-x^2}{2} + 2x\right]_0^2 + \left[\frac{x^2}{2} - 2x\right]_2^5$ (1/2)

$= \left([-2+4] - [0]\right) + \left(\left[\frac{25}{2} - 1\right] - [2-4]\right)$

$= \frac{13}{2}$ (1/2)

Q5 (a) $f'(x) = x^2 - 9 \quad \therefore x = \pm 3$



$\therefore f(x)$ is decreasing at $x \in]-3, 3[$ $\frac{1}{2}$

, $f(x)$ is increasing at $x \in]-\infty, -3[\cup]3, \infty[$ $\frac{1}{2}$

$\therefore f(3) = 9 - 27 + 3 = -15$ local min. value $\frac{1}{2}$

$\therefore f(-3) = -9 + 27 + 3 = 21$ local max. value $\frac{1}{2}$

(2 marks)

b)

1) $\int \frac{\cos^3 x - 5}{\cos^2 x}\, dx = \int (\cos x - 5\sec^2 x)\, dx$ $\frac{1}{2}$

$= \sin x - 5\tan x + c$ $\frac{1}{2}$

2) put $y = \ln 5x \quad dy = \frac{1}{x}\, dx$ $\frac{1}{2}$

$\therefore \int y\, dy = \frac{1}{2} y^2 + c$

$= \frac{1}{2} (\ln 5x)^2 + c$ $\frac{1}{2}$

(2 marks)

Another answer

$\int \ln 5x \; \frac{dx}{x}$ $\frac{1}{2}$ $= \frac{(\ln 5x)^2}{2} + c$ $\frac{1}{2}$

(The function) (its derivative)