

نموذج رقم (١)

الأزهر الشريف

قطاع المعاهد الأزهرية

نموذج إجابة لامتحان الشهادة الثانوية الأزهرية

للعام الدراسي ١٤٤٠ هـ - ٢٠١٨ / ٢٠١٩ م

الدور الثاني

القسم : العلمي (نظام حديث)

مادة : الجبر والهندسة الفراغية (مترجم)

عدد الأسئلة (٥)

علماً بأن النموذج استرشادياً

Q1

(3 marks)

	Solution	mark
1	120	$\triangle \frac{1}{2}$
2	10	$\triangle \frac{1}{2}$
3	100	$\triangle \frac{1}{2}$
4	$\left\{ 16 - \frac{13}{5} \right\}$	$\triangle \frac{1}{2}$
5	$20 + 10w^2$	$\triangle \frac{1}{2}$
6	-1	$\triangle \frac{1}{2}$

Question (2) [2 marks]

$$(a) \therefore T_{r+1} = {}^9C_r \left(\frac{-1}{2x}\right)^r (2x)^{9-r}$$

$$= {}^9C_r \left(\frac{-1}{2}\right)^r (2)^{9-r} (x)^{9-3r} \quad \triangle \frac{1}{2}$$

$$\text{Let } 9 - 3r = 3 \quad \therefore r = 2$$

$$\therefore \text{Coefficient of } x^3 = \text{Coefficient of } T_3$$

$$= {}^9C_2 \left(\frac{-1}{2}\right)^2 (2)^7 = 1152 \quad \triangle \frac{1}{2}$$

\therefore The order of the two middle terms are 5, 6 $\triangle \frac{1}{2}$

$$\therefore \frac{T_6}{T_5} = 1 \quad \therefore \frac{9-5+1}{5} \times \frac{-1}{2x^2} \times \frac{1}{2x} = 1$$

$$\therefore x = \sqrt[3]{\frac{-1}{4}} \quad \triangle \frac{1}{2}$$

$$(b) \therefore \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 2 \\ 3 & 4 & -1 \end{vmatrix} \quad \triangle \frac{1}{2} \quad [2 \text{ marks}]$$

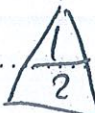
$$= -9\hat{i} + 3\hat{j} - 15\hat{k} \quad \triangle \frac{1}{2}$$


$$\therefore \|\vec{A} \times \vec{B}\| = \sqrt{(-9)^2 + (3)^2 + (-15)^2} = 3\sqrt{35} \quad \triangle \frac{1}{2}$$


\therefore area of parallelogram = $\|\vec{A} \times \vec{B}\|$


$$= 3\sqrt{35} \text{ unit of area} \quad \triangle \frac{1}{2}$$

Question (3) [2 marks]


(a) $\therefore x = 1 \Rightarrow 0 \quad \therefore x = 1$ 

$\therefore \begin{vmatrix} 1 & 2 & 2 \\ 1 & 1 & 1 \\ 3 & 4 & k \end{vmatrix} = \text{Zero}$ 

$\therefore (k-4) - 2(k-3) + 2(4-3) = \text{Zero}$ 


$\therefore \boxed{k = 4}$ 

(2 marks)


(b) $\vec{d}_1 = \overrightarrow{BC} = \vec{C} - \vec{B} = (5, -1, -1)$ 

\therefore The two straight lines are parallel

$\therefore \vec{d} = \vec{d}_1 = (5, -1, -1)$

The vector equation $\vec{r} = (1, -1, 0) + t(5, -1, -1)$ 

$\therefore x = 1 + 5t, y = -1 - t, z = -t$ Parametric eqⁿ



$\frac{x-1}{5} = \frac{y+1}{-1} = \frac{z}{-1}$ Cartesian eqⁿ 


$\therefore \frac{-14-1}{5} = \frac{2+1}{-1} = \frac{3}{-1}$

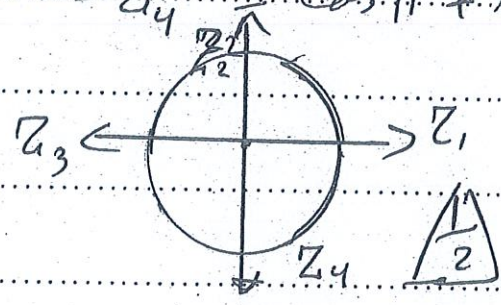
\therefore The point $(-14, 2, 3)$ lies on the st. line




Question (4) [2 marks]


(a) $\therefore Z^4 = 1 \therefore Z^4 = \cos 0^\circ + i \sin 0^\circ$ 
 $\therefore Z = \cos \left(\frac{0 + 2\pi r}{4} \right) + i \sin \left(\frac{0 + 2\pi r}{4} \right)$ 
 $r = 0, 1, 2, 3$


at $r = 0 \therefore Z_1 = \cos 0^\circ + i \sin 0^\circ$
 at $r = 1 \therefore Z_2 = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$
 at $r = 2 \therefore Z_3 = \cos \pi + i \sin \pi$
 at $r = 3 \therefore Z_4 = \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}$ 




(b) $\therefore \theta_x = \theta_y = \theta_z$ [2 marks]

$\therefore \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$ 

$\therefore 3 \cos^2 \theta_x = 1 \therefore \cos \theta_x = \pm \frac{1}{\sqrt{3}}$ 

$\therefore \vec{A} = 7\sqrt{3} \left(\pm \frac{1}{\sqrt{3}} \hat{i}, \pm \frac{1}{\sqrt{3}} \hat{j}, \pm \frac{1}{\sqrt{3}} \hat{k} \right)$ 

$\therefore \vec{A} = \pm 7 (\hat{i} + \hat{j} + \hat{k})$ 

Question (5) [2 marks]

$$(a) R.H.S = \frac{1}{abc} \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ abc & bca & abc \end{vmatrix} \begin{array}{c} \triangle \\ 1 \\ 2 \end{array}$$

$$= \frac{abc}{abc} \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix} \begin{array}{c} \triangle \\ 1 \\ 2 \end{array}$$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix} = L.H.S \begin{array}{c} \triangle \\ 1 \\ 2 \end{array}$$

[2 marks]

$$(b) \therefore \cos \theta = \frac{|\vec{d}_1 \cdot \vec{d}_2|}{\|\vec{d}_1\| \|\vec{d}_2\|} \begin{array}{c} \triangle \\ 1 \\ 2 \end{array}$$

$$\therefore \cos \theta = \frac{6}{\sqrt{6} \times \sqrt{24}} \begin{array}{c} \triangle \\ 1 \\ 2 \end{array}$$

$$= \frac{1}{2} \begin{array}{c} \triangle \\ 1 \\ 2 \end{array}$$

$$\therefore \theta = 60^\circ \begin{array}{c} \triangle \\ 1 \\ 2 \end{array}$$